RESEARCHERID
THOMSON REUTERS

[COTII- 2018] ISSN 2348 – 8034
Impact Factor- 5.070

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

COMPARATIVE ANALYSIS OF SOLUTION METHODS OF ASSIGNMENT PROBLEM

Dr. PK Dwivedi[1], Suresh Maithani[2] & Ajay Kumar Mishra[3]
[1&2]Ambalika Institute of Management & Technology, Lucknow, UP
[3]Research Scholar SunRise University, Alwar, Rajasthan

ABSTRACT

In the present paper our aim is to develop a new method for solving the Assignment Problem, which will be known as **Matrix one's division method**. Further we have given the numerical example & the solution by above method and then compare with the Hungarian method. We found that results are same in both cases. In another situation result is different but computational procedure is very less.

Keywords : Assignment Problem, Linear Programming, Matrix-one' division.

I. INTRODUCTION

– Assignment problem is the subclass of linear Programming problem, in which the objective is to assign a number of origins (jobs) to equal number of destinations (persons) at a minimum cost or maximum profit [1] [2] [3] [4]. Suppose there are n jobs to be performed and n-persons are available for doing these jobs. Assume that each person can do each job at a time through varying degree of efficiency. Let Cij be the cost if ith person is assigned to the jth job. The problem is to find an assignment such that the total cost of performing all jobs is .The Assignment problem can be stated in the form of n×n cost matrix [Cij] of real numbers as given in the following table :

Jobs

1 2 3……………..j …………………n

$$\text{Persons}\quad \begin{matrix}1\\2\\3\\i\\n\end{matrix}\begin{bmatrix} C_{11}x_{11} & C_{12}\,x_{12} & C_{13}\,x_{13}\ldots\ldots & C_{1n}\,x_{1n} \\ C_{21}x_{21} & C_{22}\,\,x_{22} & C_{23}\,x_{23}\ldots\ldots & C_{2n}x_{2n} \\ C_{31}x_{31} & C_{32}\,\,x_{32} & C_{33}\,x_{33}\ldots\ldots & C_{3n}x_{3n} \\ C_{i1}x_{i1.} & C_{i2}\,\,x_{i1} & C_{i3}\,x_{i3}\ldots\ldots & C_{in}\,x_{in} \end{bmatrix}_{n\times n}$$

Mathematically an assignment problem can be stated as

$$\text{Min } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}\,x_{ij}$$

Subject to the restrictions

$$X_{ij} = \begin{cases} 1, if\ i^{th} person\ is\ assigned\ to\ j^{th}\ job \\ \qquad\qquad 0, if\ not \end{cases}$$

$\sum_{j=1}^{n} x_{ij} = 1$ – one job is done by i^{th} person

&

$\sum_{i=1}^{n} x_{ij} = 1$ – only one person should be assigned the j^{th} job.

Many researcher have contributed the work on Assignment model [7] [8] [9] [10] [11]. To find the solution of Assignment model in general we use Hungarian method. In the present paper, we have revised the algorithm of Hungarian method and then solve the problem. We see that the final output is same. Further we have made comparative study in Hungarian method and revised method.

II. PROPOSED NEW METHOD FOR SOLVING ASSIGNMENT PROBLEM –

This section present a new method to solve the assignment problem, which is different from Hungarian method. We call it Matrix one's Division Method. It is based on creating some ones in assignment matrix and try to find the complete assignment in terms of ones. Many Authors have solved the Assignment problem by Matrix one method [11] [12] [13].

(a) Proposed Algorithm – The solution of Assignment problem can be obtained by using following revised Algorithm.

1- Prepare a cost matrix. If the cost matrix is not a square matrix, then make it square by adding dummy row or dummy column with zero cost.

2- Select minimum/maximum element in each row. Divide all elements of the row by minimum element and find one's and further determine the revised matrix.

3- Further modify the resulting matrix by selecting minimum element in each column & then divide all elements of column by minimum element of each column to find one's and determine the new modified matrix.

4- Draw minimum number of horizontal & vertical lines to cover all the one's. Let us consider the number of lines be N and order of matrix be n. If N=n, then we can make the Assignment, otherwise go the next step.

5- Determine the smallest uncovered element in matrix. Divide all uncovered element by this minimum element to find various ones Also fine next modified matrix. If the number of horizontal & vertical lines are equal to n i.e. order of matrix, otherwise repeat the process until we get required request.

6- To make the assignment examine the row with single one's, If row wise single one is found then circle it and cross all one's in corresponding column's because they can not be selected for assignment. Continue the process until all the one's are examined. We can repeat the same process for column also.

7- Repeat above steps successively until one of the following situation arises.

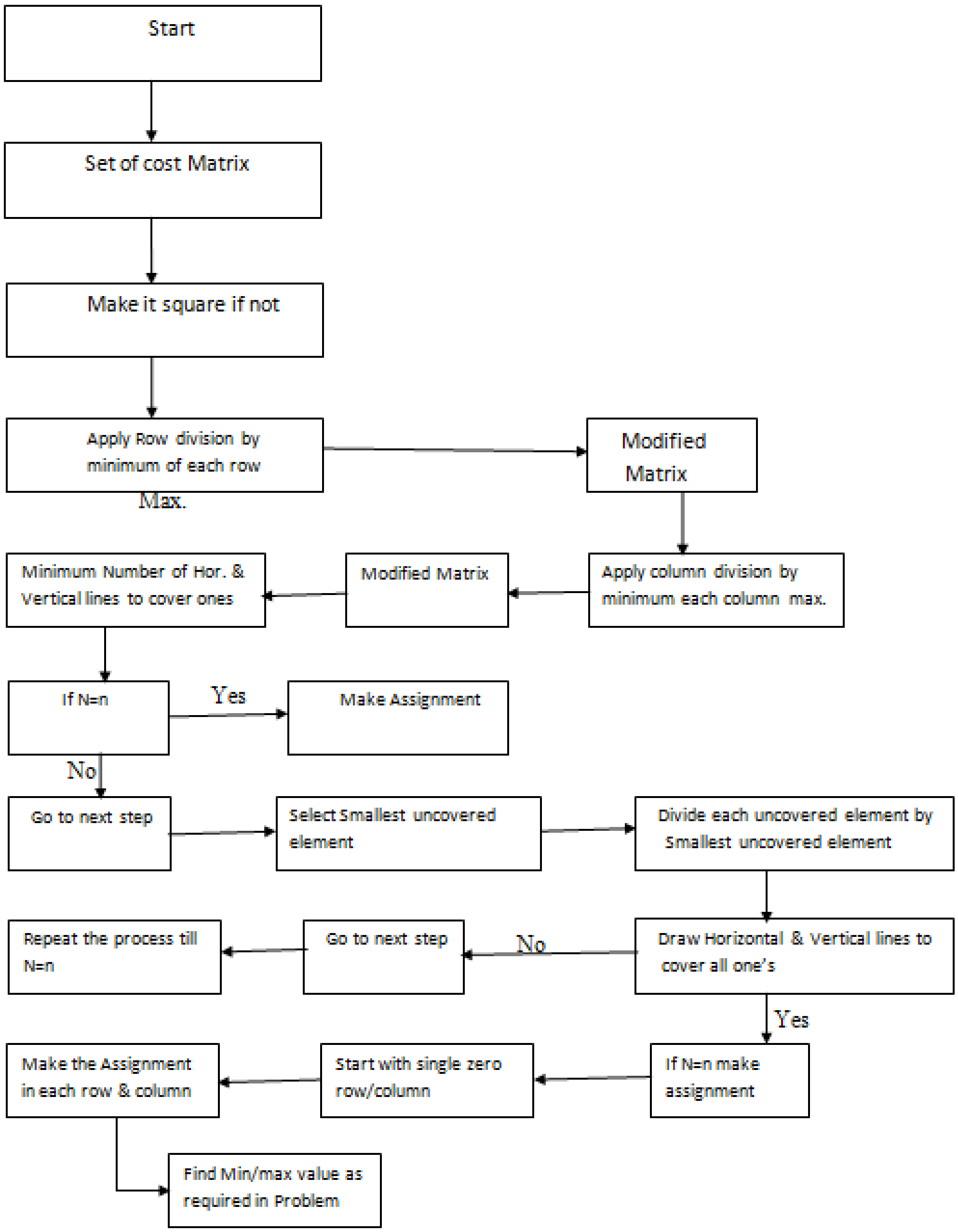
(i) If no unmarked one is left, then the process ends or

(ii) If these lies more than one unmarked one's in any columns or row, then circle one of the unmarked one's arbitrarily and mark a cross on remaining one's of corresponding column. Repeat the process until no unmarked one is left.

8- Thus exactly one marked circle 'one' in each row & each column of the matrix is obtained. The assignment corresponding to these marked circled one will give the optimal Assignment.

(b) Flow chart for Matrix one's Division Method.

ISSN 2348 – 8034
Impact Factor- 5.070



3- Numerical Example by Matrix one's Division Method

Core- I

Example – 1 Solve the following Assignment Problem

$$\begin{bmatrix} 5 & 3 & 2 & 8 \\ 7 & 9 & 2 & 6 \\ 6 & 4 & 5 & 7 \\ 5 & 7 & 7 & 8 \end{bmatrix}$$

Solution –

1- Find minimum element of each row and write it on Right hand side of matrix.

Min (a_i)

ISSN 2348 – 8034
Impact Factor- 5.070

$$\begin{bmatrix} 5 & 3 & 2 & 8 \\ 7 & 9 & 2 & 6 \\ 6 & 4 & 5 & 7 \\ 5 & 7 & 7 & 8 \end{bmatrix} \begin{matrix} 2 \\ 2 \\ 4 \\ 5 \end{matrix}$$

Now divide each element of each row by minimum of each row to obtain one's & we have the modified matrix.

$$\begin{bmatrix} \frac{5}{2} & \frac{3}{2} & 1 & 4 \\ \frac{7}{2} & \frac{9}{2} & 1 & 3 \\ \frac{3}{2} & 1 & \frac{5}{4} & \frac{7}{4} \\ 1 & \frac{7}{5} & \frac{7}{5} & \frac{8}{5} \end{bmatrix}$$

2- Find minimum element of each column in above modified matrix and write below the corresponding column, we have

$$\begin{bmatrix} \frac{5}{2} & \frac{3}{2} & 1 & 4 \\ \frac{7}{2} & \frac{9}{2} & 1 & 3 \\ \frac{3}{2} & 1 & \frac{5}{4} & \frac{7}{4} \\ 1 & \frac{7}{5} & \frac{7}{5} & \frac{8}{5} \end{bmatrix}$$

Min bj 1 1 1 $\frac{8}{5}$

Now divide each element of each column by minimum element to find one's we have

$$\begin{bmatrix} \frac{5}{2} & \frac{3}{2} & 1 & \frac{5}{2} \\ \frac{7}{2} & \frac{9}{2} & 1 & \frac{15}{2} \\ \frac{3}{2} & 1 & \frac{5}{4} & \frac{35}{32} \\ 1 & \frac{7}{5} & \frac{7}{5} & 1 \end{bmatrix}$$

Here N= 3<n=4

3- Select the minimum uncovered element and divide all uncovered element by minimum uncovered element, we have

ISSN 2348 – 8034
Impact Factor- 5.070

$$\begin{bmatrix} \frac{16}{7} & \frac{3}{2} & 1 & \frac{16}{7} \\ \frac{16}{5} & \frac{9}{2} & 1 & \frac{48}{7} \\ \frac{48}{35} & 1 & \frac{5}{4} & 1 \\ 1 & \frac{7}{5} & \frac{7}{5} & 1 \end{bmatrix}$$

Further N=3 < n=4, we go to next step

4- Select the next minimum uncovered element and repeat the above process, we have

$$\begin{bmatrix} \frac{32}{21} & 1 & 1 & \frac{16}{7} \\ \frac{35}{15} & 3 & 1 & \frac{48}{7} \\ \frac{48}{35} & 1 & \frac{5}{4} & 1 \\ 1 & \frac{7}{5} & \frac{7}{5} & 1 \end{bmatrix}$$

Here N= 4 = n so, we can make Assignment

Person	Job	Cost
1	2	3
2	3	2
3	4	7
4	1	5
		17

Example -2 **Solve the following Assignment problem**

$$\begin{bmatrix} \mathbf{5} & \mathbf{3} & \mathbf{2} & \mathbf{8} \\ \mathbf{7} & \mathbf{9} & \mathbf{2} & \mathbf{6} \\ \mathbf{6} & \mathbf{4} & \mathbf{5} & \mathbf{7} \\ \mathbf{5} & \mathbf{7} & \mathbf{7} & \mathbf{8} \end{bmatrix}$$

Solution- 1- Write the maximum element of each row on RHS of Matrix

Max (a_i)

$$\begin{bmatrix} 5 & 3 & 2 & 8 \\ 7 & 9 & 2 & 6 \\ 6 & 4 & 5 & 7 \\ 5 & 7 & 7 & 8 \end{bmatrix} \begin{matrix} 8 \\ 9 \\ 7 \\ 8 \end{matrix}$$

Now divide each element of each row by maximum of each row to obtain one's and find the next modified matrix.

ISSN 2348 – 8034
Impact Factor- 5.070

$$\begin{bmatrix} \frac{5}{8} & \frac{3}{8} & \frac{1}{4} & 1 \\ \frac{7}{9} & 1 & \frac{2}{9} & \frac{2}{3} \\ \frac{6}{7} & \frac{4}{7} & \frac{5}{7} & 1 \\ \frac{5}{8} & \frac{7}{8} & \frac{7}{8} & 1 \end{bmatrix}$$

2- Find maximum element in each column & write it bottom of that column; we have

$$\begin{bmatrix} \frac{5}{8} & \frac{3}{8} & \frac{1}{4} & 1 \\ \frac{7}{9} & 1 & \frac{2}{9} & \frac{2}{3} \\ \frac{6}{7} & \frac{4}{7} & \frac{5}{7} & 1 \\ \frac{5}{8} & \frac{7}{8} & \frac{7}{8} & 1 \end{bmatrix}$$

Max bj - $\frac{6}{7}$ 1 $\frac{7}{8}$ 1

3- Dividing each element of each column by maximum element of that column and find the next modified matrix.

$$\begin{bmatrix} \frac{35}{48} & \frac{3}{8} & \frac{2}{7} & 1 \\ \frac{49}{54} & 1 & \frac{16}{63} & \frac{2}{3} \\ 1 & \frac{4}{7} & \frac{40}{49} & 1 \\ \frac{35}{48} & \frac{7}{8} & 1 & 1 \end{bmatrix}$$

Here N=n=4, Hence we can make the Assignment

4- Now Give Assignment with single 1, we have and mark all arrangement, we have

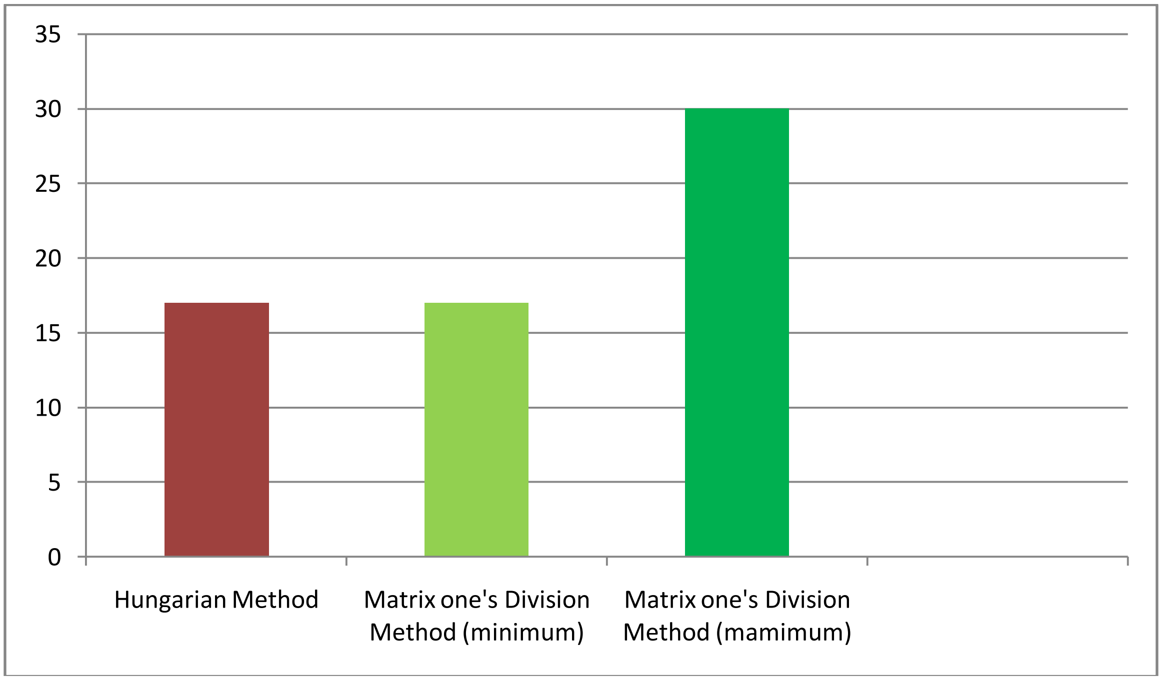
Person	Job	Cost
1	4	8
2	2	9
3	1	6
4	3	7
		Rs. 30

Ex. 3 Solve the Assignment problem by Hungarian method

$$\begin{bmatrix} \mathbf{5} & \mathbf{3} & \mathbf{2} & \mathbf{8} \\ \mathbf{7} & \mathbf{9} & \mathbf{2} & \mathbf{6} \\ \mathbf{6} & \mathbf{4} & \mathbf{5} & \mathbf{7} \\ \mathbf{5} & \mathbf{7} & \mathbf{7} & \mathbf{8} \end{bmatrix}$$

Solution - Solution by Hungarian method is given [14].

4- **Comparison of values obtained by various methods**



In tabular form it is given by

Problem	Hungarian Method	Matrix Division Method	Optimum Value
1	17	17, 30	17

III. CONCLUSION

In this paper a new & simple approach has been introduced for solving Assignment problem. The new method is based on creating some one's in matrix and find an assignment interns of one's. Finally we observe that HA method and matrix division method with minimum a_i's provide the same cost of Assignment but matrix division method with maximum a_i's gives the different values but computational procedure is very less in comparison to above methods. Hence this paper attempts to propose a method for solving Assignment problem, which is different from other methods

REFERENCES

1. *H.W.Kuhn, "The Hungarian Method for the Assignment Problem", Naval Research Logistic Quarterly, 2 (1995, pp. 83-97).*
2. *A. kumar "A Modified Method for solving the Unbalanced Assignment Problems", Applied Mathematics and computation, 176,2006, pp.76-82.*
3. *Anupong Iampang, Veera Boonjing and Pisit Chanvarasuth, "A cost and space efficient method for unbalanced assignment problems",978-1-4244-8503-1/10 2010 IEEE.*
4. *A.Ahmed and Afaq Ahmad "A new method for finding an optimal solution of assignment problem",IJMMS,2014,12(1):10-15 ISSN:2166-286X.*
5. *Hadi Basirzadeh "Ones Assignment Method for solving Assignment problems",AMS, vol. 6, 2012, no. 47,2345-2355.*

6. *Anju Khandelwal "A Modified Approach For Assignment Method",IJLRST,vol:3, Issue 2: Page No.136-138, March-April, 2014,ISSN:2278-5299.*
7. *A. Schrijver, combinatorial optimization: polyhedra and efficiency, vol. A. Paths, Flows, Matchings, Springer, Berlin, 2003.*
8. *Hadi Basirzadeh, Vahid Morovati, Abbas Sayadi "A quick method to calculate the super-efficient point in multi-objective assignment problems" ,Journal of mathematics and computer science", p157-162,.2014.*
9. *Dr.A.Ramesh Kumar and S. Deepa "Restrictions of interval Assignment problem using Hungarian method" vol.2 Issue-12 (December 2014), ISSN:2321-1776.*
10. *Jatinder Pal Singh , Neha Ishesh Thakur " A Novel Method To Solve Assignment Problem in Fuzzy Environment", IISTE, ISSN :2224-6096, vol. no.2, 2015.*
11. *Taejoon Kim,Member, IEEE, and Miaomiao Dong "An Iterative Method to joint Relay Selection and Resource Allocation for D2D Communications", IEEE,Vol.3No.6,December 2014.*
12. *Jameer. G. Kotwal, Tanuja S. Dhope Computer Department Gyanchand Hirachand Raisoni College of Engineering, Pune, Maharashtra, India International Journal of Advanced Research in Computer Science and Software Engineering.*
13. *Volume 5, Issue 7, July 2015 ISSN: 2277 128X.*
14. *Manohar Mahajan, Operation Research Dhanpat Rai & Co. Private Ltd. Delhi Reprint 2016*